Solution to 2024-JEE Advanced Full Test-1 |Paper-2

PHYSICS

1.(6) Time of flight (T)

$$S = ut + \frac{1}{2}at^2$$

$$-20 = 0 - \frac{1}{2}gT^2$$

$$T = 2\sec$$

Range (R)

$$R = 5T$$

$$R = 10$$

$$R = D + D - x_0$$

$$10 = 2D - x_0$$

$$10 = 16 - x_0$$

$$x_0 = 6m$$

2.(5) Let the SHM equations for the two particles be given by

$$x_1 = 5\sin\left(\frac{2\pi}{6}(t+1)\right)$$

$$x_2 = 5\sin\left(\frac{2\pi}{6}t\right)$$

$$|x_2 - x_1| = 5\left[\sin\left(\frac{\pi}{3}t + \frac{\pi}{3}\right) - \sin\left(\frac{\pi t}{3}\right)\right]$$

$$\frac{d|x_2 - x_1|}{dt} = \frac{5\pi}{3}\cos\left(\frac{\pi t}{3} + \frac{\pi}{3}\right) - \frac{5\pi}{3}\cos\left(\frac{\pi t}{3}\right)$$

$$0 = \cos\left(\frac{\pi t}{3} + \frac{\pi}{3}\right) = \cos\left(\frac{\pi t}{3}\right)$$

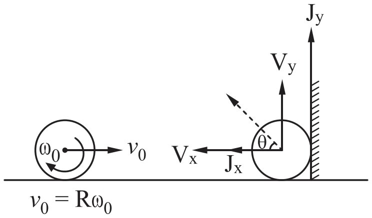
$$\Rightarrow \quad \frac{\pi t}{3} + \frac{\pi}{3} = 2n\pi \pm \frac{\pi t}{3} \qquad \Rightarrow \quad \frac{\pi t}{3} + \frac{\pi}{3} = 2\pi - \frac{\pi t}{3} \qquad [\text{Putting } n = 1]$$

$$\Rightarrow \quad \frac{2\pi t}{3} = 2\pi - \frac{\pi}{3} \qquad \Rightarrow \quad 2t = 6 - 1 \qquad \Rightarrow \quad t = \frac{5}{2}$$

$$\therefore \quad |x_2 - x_1| = \left|5\left[\sin\left(\frac{5\pi}{6} + \frac{\pi}{3}\right) - \sin\left(\frac{5\pi}{6}\right)\right]\right| \qquad \therefore \quad = \left|5\left(-\sin\frac{\pi}{6} - \sin\frac{\pi}{6}\right)\right|$$

$$= 5 \times 2 \times \sin\frac{\pi}{6} = 5 \times 2 \times \frac{1}{2} = 5cm$$

3.(5) Since collision is elastic



$$e = 1 = \frac{V_x}{v_0} \Rightarrow V_x = v_0$$

$$\therefore \quad J_x = 2mv_0$$

Also $J_y = \mu \int N dt = \mu J_x = mV_y$

$$\therefore \quad v_y = \frac{\mu \times 2mv_0}{m} = 2\mu v_0 = v_0$$

$$R = \frac{2v_0^2}{g} = 5m$$

4.(3) For x_1

$$\overset{\longleftarrow x_1 \longrightarrow}{\overset{B}{\square} \rightarrow 30ms^{-1} \quad \overset{A}{\square} \rightarrow 30ms^{-1}}$$

If collision does not take place, the final velocities of both should be same.

$\therefore \quad v_f = 30 - \frac{30}{7}t = 30 - 3(t-1)$ $\therefore$ $\frac{30}{7}t = 3t - 3$ or $\frac{9t}{7} = -3 \Rightarrow t = -\frac{21}{9}$

The negative of t indicates that both comes in rest before collision. The distance moved by bus B before coming to rest is $s_1 = \frac{30^2}{2\times 3} + 30 \times 1 = 180\ m$

The distance moved by bus A before coming to rest is $s_2 = \frac{30^2}{2\times\frac{30}{7}} = \frac{30 \times 7}{2} = 105\ m$

$$\therefore \quad x_1 = s_1 - s_2 = 180 - 105 = 75\ m$$

$$\underset{\longleftarrow x_2 \longrightarrow}{\overset{A}{\square} \rightarrow 30m/s \quad \overset{B}{\square} \rightarrow 30m/s}$$

For x_2 : $v_f = 30 - \frac{30}{7}(t-1) = 30 - 3t$ or $\frac{30}{7}t - \frac{30}{7} = 3t$ or $\frac{9}{7}t = \frac{30}{7}$ $\therefore$ $t = \frac{10}{3}s$

The positive value of *t* indicates that before collision, velocities of both buses are non-zero but same.

So, the distance moved by A before collision is $s_1 = 30 \times 1 + 30 \times \left(\frac{10}{3} - 1\right) - \frac{1}{2} \times \frac{30}{7}\left(\frac{10}{3} - 1\right)^2$

$$s_1 = \frac{265}{3}m$$

Distance moved by B before collision is : $s_2 = 30 \times \frac{10}{3} - \frac{1}{2} \times 3 \times \left(\frac{10}{3}\right)^2 = 100 - \frac{50}{3} = \frac{250}{3}$

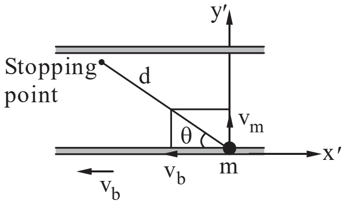
$$x_2 = s_1 - s_2 = \frac{265}{3} - \frac{250}{3} = \frac{15}{3} = 5m \qquad \text{or} \qquad \frac{x_1}{5x_2} = \frac{75}{25} = 3$$

5.(5) $\vec{v}_{m,b} = \vec{v}_m - \vec{v}_b \,...(i)$

Applying equation (1) to x-and y-components of velocity, we obtain

$(v_{mb})_x = -v_b$ and $(v_{mb})_y = v_m$

From work-energy theorem in (x', y') coordinate system ,



$\Delta W = \Delta KE$

$-\mu mgd = -\frac{1}{2}m(v_m^2 + v_b^2)$ where d is the stopping distance

$$d = \frac{1}{2\mu g}(v_m^2 + v_b^2)$$

In the (x', y') coordinate system

$$x' = -d\cos\theta = -d\frac{v_b}{\sqrt{v_m^2 + v_b^2}} = \frac{-v_b\sqrt{v_m^2 + v_b^2}}{2\mu g}$$

$$y' = d\sin\theta = d\frac{v_m}{\sqrt{v_m^2 + v_b^2}} = \frac{v_m}{2\mu g}\sqrt{v_m^2 + v_b^2}$$

The particle has a constant retardation of magnitude μg in both the reference frames, as these are inertial. Therefore time taken to stop can be determined from the equation.

$$0 = \left[\sqrt{v_m^2 + v_b^2}\right] - \mu g t$$

$$t = \frac{1}{\mu g}\sqrt{v_m^2 + v_b^2}$$

We may convert the stopping distance to a fixed coordinate system by the following equations

$x = x' + v_b t, y = y'$

$$x = \frac{v_b}{2\mu g}\sqrt{v_m^2 + v_b^2} \quad ; \quad y = \frac{v_m}{2\mu g}\sqrt{v_m^2 + v_b^2}$$

Now, $xy = \frac{1}{5} \quad \Rightarrow \quad \frac{1}{xy} = 5$

6.(6) Power absorbed by earth = power emitted by earth.

$$\Rightarrow \quad \frac{e\sigma(4\pi R_s^2)T_s^4}{4\pi r^2} \times \pi R_e^2 = e\sigma(4\pi R_e^2)T_e^4 \quad \Rightarrow \quad T_e = T_s\sqrt{\frac{R_s}{2r}} = T_s\sqrt{\frac{R_s}{2\times 200\,R_s}}$$

$$\Rightarrow \quad T_e = \frac{T_s}{20} = 300\ K \qquad \therefore \qquad n = 6$$

7.(3) Here given

$\alpha_{glass} = 9\times10^{-6}K^{-1} \qquad \therefore \qquad \gamma_{glass} = 27\times10^{-6}K^{-1}$

$= 0.27\times10^{-4}K^{-1}$

And $\gamma_{mercury} = 1.8\times10^{-4}K^{-1}$ Now, $\Delta V_{glass} = \Delta V_{air} + \Delta V_{mercury}$

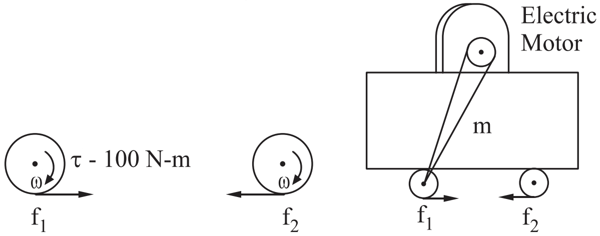
Given $\Delta V_{air} = 0 \qquad \therefore \qquad \Delta V_{glass} = \Delta V_{mercury}$

$$\Rightarrow \quad \gamma_{glass}\Delta t \times V_{glass} = V_{mercury} \times \gamma_{mercury} \times \Delta t \quad \Rightarrow \quad V_{mercury} = \frac{\gamma_{glass}}{\gamma_{mercury}} \times V_{glass}$$

$\Rightarrow \quad \dfrac{\gamma_{glass}}{\gamma_{mercury}} = \dfrac{x}{20}$ [Given $V_{mercury} = \dfrac{x}{20} V_{glass}$]

$\Rightarrow \quad \dfrac{0.27 \times 10^{-4} \times 20}{1.8 \times 10^{-4}} = x \qquad \Rightarrow \quad x = \dfrac{5.4}{1.8} = 3$

8.(5) Now F.B.D. of each cylinder could be shown as



$\tau - f_1 R = I\alpha$

$\Rightarrow \quad \tau - f_1 R = I\dfrac{a}{r}$ . . . (i)

$f_2 R = I\alpha$

$\Rightarrow \quad f_2 R = I\dfrac{a}{r}$ . . . (ii)

$f_1 - f_2 = ma$

Putting the given values, we get

$100 - f_1 = 0.5a$

$f_2 = 0.5a$

$f_1 - f_2 = 3a$

Solving these three equations for a we get

$a = 25 m/s^2 \quad \Rightarrow \quad a = 5 \times 5 \, m/s^2$

$\therefore \quad n = 5$

9.(AD) Suppose blocks A and B move together. Applying NLM on C, A + B and D respectively

$60 - T = 6a$

$T - 18 - T' = 9a \qquad T' - 10 = 1a$

Solving $\quad a = 2 m/s^2$

To check slipping between A and B, we have to find friction force in this case. If it is less than limiting static friction, then there will be no slipping between A and B.

Applying NLM on A

$T - f = 6 \times 2$

As $\quad T = 48N \qquad f = 36N$

And $\quad f_{x\max.} = 42N$ hence A and B move together.

10.(ABD) $-\frac{GMm}{R}+0=\frac{3}{2}\frac{GMm}{R}+\frac{1}{2}mv^2 \Rightarrow v=\sqrt{\frac{GM}{R}}$

$\therefore$ velocity after collision $=v'=ev=\frac{1}{2}\sqrt{\frac{GM}{R}}$ $\therefore$ $J=\frac{3}{2}m\sqrt{\frac{GM}{R}}$

Had the tunnel been across the earth, the particle would have executed SHM

Then when $y=\frac{R}{2}$

$t_1=\frac{T}{6}$

$\therefore$ time taken for the II half is $t_2=\frac{T}{4}-\frac{T}{6}=\frac{T}{12}$ $\therefore$ $\frac{t_1}{t_2}=\frac{2}{1}$

$\frac{-3}{2}\frac{GMm}{R}+\frac{1}{2}mv_e^2=0$ $\therefore$ $v_e=\sqrt{\frac{3GM}{R}}$

11.(BC) Case I : Particle moves on an ellipse

Case II : Particle performing SHM along $y=\frac{Bx}{A}$

12.(BC) $g=\frac{GM}{R^2}$

$$g_P(\text{at surface})=\frac{G(M/2)}{(4R)^2}=\frac{g}{32}$$

$$g_P(\text{above height } R)=\frac{g/32}{\left(1+\frac{R}{4R}\right)}=\frac{g}{32}\times\frac{16}{25}=\frac{g}{50}$$

$$g_P(\text{below the surface})=\frac{g}{32}\left(1-\frac{R/2}{4R}\right)=\frac{7g}{256}$$

13.(ABCD) Let acceleration of rocket be a after 1 sec, height of rocket $=\frac{1}{2}a(1)^2=\frac{a}{2}$

Velocity of rocket = a

For bolt : $U_B=a,\ a_B=-10,\ s_B=\frac{-a}{2},\ T=2$

$\frac{-a}{2}=a(2)+\frac{1}{2}(-10)(4)$; $20=\frac{5a}{2}\Rightarrow a=8m/s^2$

Fuel of the rocket is finished after 5 sec.

$h=\frac{1}{2}at^2=\frac{1}{2}(8)(25)=100m$

Speed is also maximum at this moment $V_{\max}=a(5)=40m/s$

After this, rocket undergoes free fall as fuel is exhausted. To find its time of flight after this, use

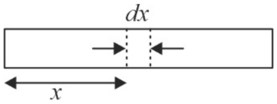
$-100=40t+\frac{1}{2}(-10)t^2$ $\Rightarrow$ $t^2-8t-20=0\Rightarrow t=10\sec$

Total air time = 5 + 10 = 15 sec.

14.(AC) New buoyant force $= V\rho(g-a)$

As the value of g effective is less than g

15.(C) Stress $= \frac{F}{A}$, is same for all points



$$Y_x = Y_0 + \frac{Y_0}{L}x$$

Let extension in dx is dy

Then $\frac{F}{A} = \left(Y_0 + \frac{Y_0}{L}x\right)\frac{dy}{dx}$

$$\int_0^L \frac{FL}{A}\cdot\frac{dx}{Y_0\left(1+\frac{x}{L}\right)} = \int_0^{\Delta L} dy \ ; \quad \frac{FL}{AY_0} ln2 = \Delta L$$

16.(C) $5 = v\cos\alpha$ **...(1)**

$v\sin\alpha = 5 + v\cos\alpha$ **... (2)**

$v\sin\alpha = 10$ **...(3)**

On squaring and adding (1) and (3)

$v^2 = 125$; $v = 5\sqrt{5}\ m/s$; $\tan\alpha = 2$; $\alpha = \tan^{-1}2$

17.(B) When plate is moving to right :

Molecules bounce back with speed $V-2u$

$\therefore$ Change in momentum of one molecule ; $\Delta P_1 = m(V-2u) - (-mV) = 2mV - 2mu$

Number of molecule hitting per unit time $n_1 = A(V-u)n$

$\therefore$ $F_1 = n_1\Delta P_1 = An(V-u)2m(V-u) = 2mnA(V-u)^2$

When plate is moving to left :

Molecule bounce back with speed $V+2u$

$\therefore$ Change in momentum of one molecule ; $\Delta P_2 = m(V+2u) - (-mV) = 2mV + 2mu$

And number of molecule hitting per unit time

$n_2 = A(V+u)\cdot n$; $F_2 = n_2\Delta P_2 = An(V+u)2m(V+u) = 2mAn(V+u)^2$

$\therefore$ $F_2 - F_1 = 2mnA[(V+u)^2 - (V-u)^2]$ $\therefore$ $8mn\ AV\cdot u$

18.(C) $\frac{dy}{dt} = \sqrt{3}\frac{dx}{dt} - 4x\frac{dx}{dt}$

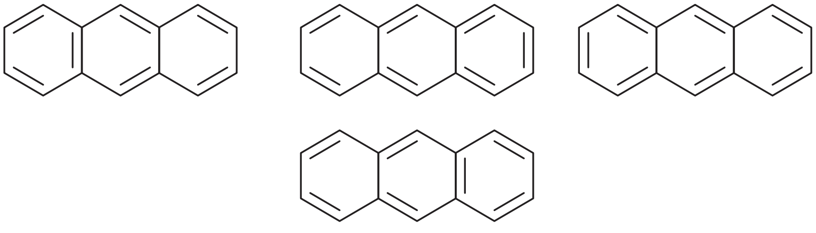
$$\frac{d^2y}{dt^2} = \sqrt{3}\frac{d^2x}{dt^2} - 4x\frac{d^2x}{dt} - 4\left(\frac{dx}{dt}\right)^2$$

$\frac{dx}{dt} = 1$,

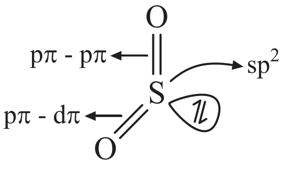
$\frac{dy}{dt} = \sqrt{3}$

CHEMISTRY

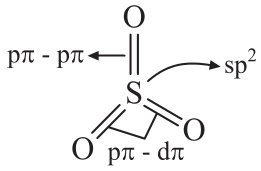
1.(4)



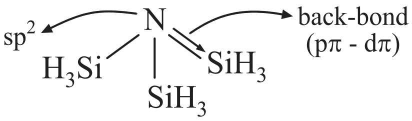
2.(3) SO_2, SO_3, $N(SiH_3)_3$



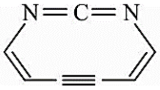
(having 1 pπ - dπ bond)



(having 2 pπ - dπ bond)

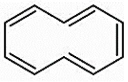


3.(6) Except



(Having sp hybrid carbon)

and



(Non aromatic)

all are aromatic compounds having sp^2 hybridized atoms.

4.(4) $N_2^+ \Rightarrow$ Bond order $= \frac{9-4}{2} = 2.5$ $\quad$ $CN^- \Rightarrow$ Bond order $= \frac{10-4}{2} = 3$

$CO \Rightarrow$ Bond order $= \frac{10-4}{2} = 3$ $\quad$ $NO^+ \Rightarrow$ Bond order $= \frac{10-4}{2} = 3$

$O_2^+ \Rightarrow$ Bond order $= \frac{10-5}{2} = 2.5$ $\quad$ $N_2 \Rightarrow$ Bond order $= \frac{10-4}{2} = 3$

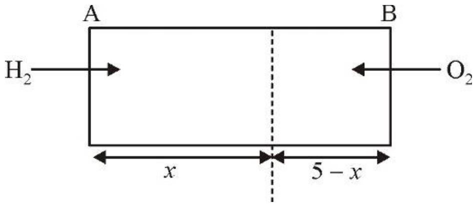
$B_2 \Rightarrow$ Bond order $= \frac{6-4}{2} = 1$ $\quad$ $N_2^- \Rightarrow$ Bond order $= \frac{10-5}{2} = 2.5$

$NO \Rightarrow$ Bond order $= \frac{10-5}{2} = 2.5$

CO, CN^-, NO^+, N_2 has bond order equal to 3

5.(6) (i) Temperature (ii) Pressure (iii) Specific heat capacity
(iv) Density (v) Molar heat capacity (vi) Molar enthalpy

6.(4) $\frac{r_{H_2}}{r_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$



$\frac{r_{H_2}}{r_{O_2}} = \frac{\frac{x}{t}}{\frac{5-x}{t}} = \frac{x}{5-x}$; $\quad 4 = \frac{x}{5-x} \Rightarrow 20 - 4x = x \Rightarrow 5x = 20 \Rightarrow$ $\quad x = 4m$

7.(4) $\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$, Z = 1 (for hydrogen atom)

$$\frac{1}{\lambda} = R_H (1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4} R_H$$

$$\lambda_{2\to 1(\text{For H-atom})} = \frac{4}{3R_H}$$

$$\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right], \quad Z = 2 \text{ (for } He^+\text{)}$$

$$\frac{1}{\lambda} = R_H (2)^2 \left[\frac{1}{2^2} - \frac{1}{n^2}\right], \quad \lambda_{n\to 2(\text{for } He^+)} = \frac{1}{4R_H \times \left[\frac{1}{2^2} - \frac{1}{n^2}\right]}$$

$$4R_H \left[\frac{1}{4} - \frac{1}{n^2}\right] = \frac{3}{4} R_H$$

$$\frac{1}{n^2} = \frac{1}{4} - \frac{3}{16} = \frac{4-3}{16} = \frac{1}{16}, \quad n = 4$$

8.(4) PCl_3F_2, CCl_4, SF_6, C_2H_4 have zero dipole moment.

9.(ABCD)

$$NaOH + NaHCO_3 \longrightarrow Na_2CO_3 + H_2O$$

1 mol 1 mol 1 mol

So we have 50 m mole of Na_2CO_3 in 50 mL solution.

With phenolphthalein : $Na_2CO_3 + HCl \longrightarrow NaHCO_3 + NaCl$

⇒ 50 mM of Na_2CO_3 can be neutralized by 50 mM of HCl. Hence statement B is correct.

With methyl orange : $Na_2CO_3 + 2HCl \longrightarrow 2NaCl + H_2CO_3$

⇒ by 50 mM of Na_2CO_3 can be neutralized by 100 mM of HCl on using methyl orange as the indicator.

With methyl orange after first end point with phenolphthalein :

$$Na_2CO_3 + HCl \xrightarrow{\text{phenolphthalein}} NaHCO_3 + NaCl \xrightarrow[\text{HCl}]{\text{methyl orange}} NaCl + H_2CO_3$$

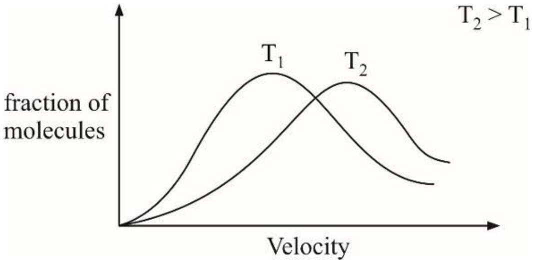
(for second step)

⇒ After achieving the phenolphthalein end point 50 mM of HCl will be sufficient to achieve the methyl orange end point.

1 mole Na_2CO_3 in 1000 ml

⇒ 50×10^{-3} moles in 50 ml solution ⇒ 50 millimoles

10.(BD)



11.(ABCD)

Given, 25 mL of $0.2\ N\ KMnO_4$ in acidic medium.

milli gm eq. of $KMnO_4 = N \times V = 0.2 \times 25 = 5$

(A) milli gm eq. $FeSO_4,\ M \times V \times n_f = 0.1 \times 50 \times 1 = 5$

(B) milli gm eq. $SnCl_2,\ M \times V \times n_f = 0.05 \times 50 \times 2 = 5$

(C) milli gm eq. $H_3AsO_3,\ M \times V \times n_f = 0.1 \times 25 \times 2 = 5$

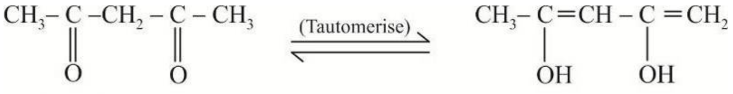
(D) milli gm eq. $H_2O_2,\ M \times V \times n_f = 0.1 \times 25 \times 2 = 5$

12.(BD) No effect of addition of inert gas at constant volume but entropy increases

13.(ABCD)

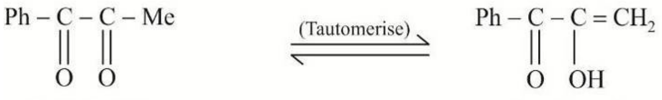
Constitutional isomers have same molecular formulae but different connectivity of functional groups or atoms. Geometrical isomers [cis-trans, E–Z are diastereomers]

14.(ABCD)



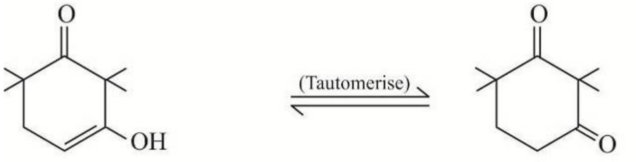
$$CH_3-\underset{\underset{O}{\|}}{C}-CH_2-\underset{\underset{O}{\|}}{C}-CH_3 \xrightleftharpoons{\text{(Tautomerise)}} CH_3-\underset{\underset{OH}{|}}{C}=CH-\underset{\underset{OH}{|}}{C}=CH_2$$

(keto form) (enol form)

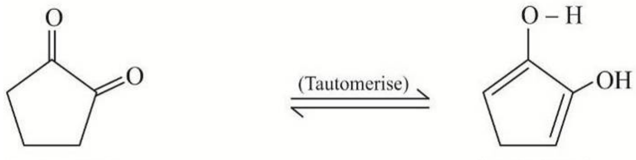


$$Ph-\underset{\underset{O}{\|}}{C}-\underset{\underset{O}{\|}}{C}-Me \xrightleftharpoons{\text{(Tautomerise)}} Ph-\underset{\underset{O}{\|}}{C}-\underset{\underset{OH}{|}}{C}=CH_2$$

(keto form) (enol form)

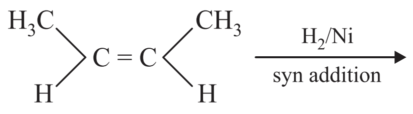


(enol form) (keto form)



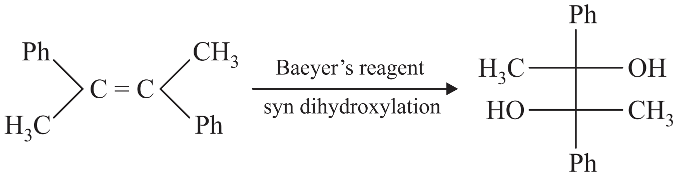
(keto form) (enol form)

15.(D) **(A)**



Product does not contain any chiral centre and hence product cannot be meso

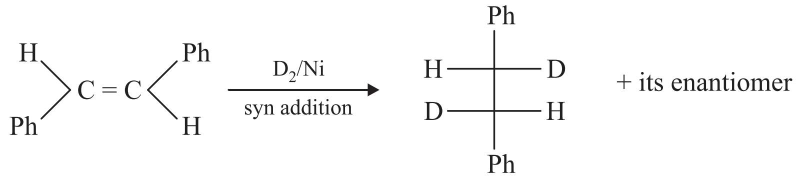
(B)



+ its enantiomer

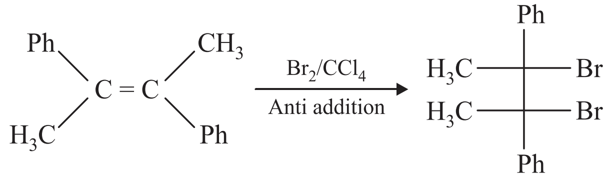
(Not a meso compound)

(C)



(Not a meso compound)

(D)



(meso compound)

16.(C) $H_2SO_4 + H_2C_2O_4 + \text{impurity} = 3.185\,g$

Let, in 1000 mL x mmole y mmole

For 10 mL solution $\frac{x}{100}$ m mole $\frac{y}{100}$ m mole

milli gm eq. of NaOH = milli gm eq. of H_2SO_4 + milli gm eq. of $H_2C_2O_4$

$$3 \times 0.1 = \frac{2x}{100} + \frac{2y}{100}$$

$$2x + 2y = 0.3 \times 100$$

$$2x + 2y = 30 \qquad \text{...... (i)}$$

For 100 mL solution

milli gm eq. of $KMnO_4$ = milli gm eq. of $H_2C_2O_4$

$$K\overset{+7}{Mn}O_4 \longrightarrow Mn^{2+} \qquad n_f = 5$$

$$H_2\overset{+3}{C_2}O_4 \longrightarrow \overset{+4}{C}O_2 \qquad n_f = 2$$

$$4 \times 0.02 \times 5 = \frac{2y}{10} \quad \Rightarrow \quad y = 2$$

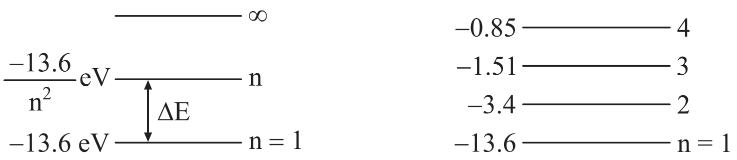
On putting the value of y in eq. (i)

$$2x + 4 = 30 \quad \Rightarrow \quad x = 13$$

milimoles of $H_2SO_4 = 13$

Mass of $H_2SO_4 = 13 \times 10^{-3} \times 98\,g$ Mass % of $H_2SO_4 = \frac{13 \times 10^{-3} \times 98}{3.185} = 40\%$

17.(D)



$\Delta E = 12.75\,eV$; $E_{n_2} - E_{n_1} = 12.75$; $E_4 - E_1 = 12.75$, so $n = 4$

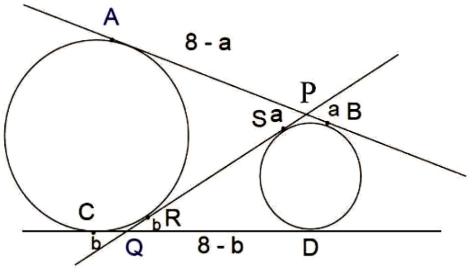
18.(A) Oxidation state of S in $H_2SO_4 = +6$, $H_2SO_3 = +4$, $H_2S = -2$

So order of oxidation state of S : $H_2SO_4 > H_2SO_3 > H_2S$

MATHEMATICS

1.(8) $PA = PR = 8 - a$

So $PQ = 8 - a + b$



Also $QD = QS = 8 - b$

So $QP = 8 - b + a$

$8 - a + b = 8 - b + a$

$\Rightarrow\ a = b$

$PQ = 8$

2.(8) $S_n = 1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} \ldots\ldots\ldots\ldots = \sum \frac{3^n - 1}{(3-1)n!}$

$= \frac{1}{2}\sum_{r=1}^{n} \frac{3^r - 1}{r!} = \frac{1}{2}\left[\sum \frac{3^r}{r!} - \frac{1}{r!}\right]$

$\lim_{n\to\infty}(S_n) = \frac{1}{2}\left[\left(e^3 - 1\right) - (e - 1)\right] = \frac{1}{2}\left[e^3 - e\right] \approx 8.59$

$[S_n] = 8$

3.(1) Equation of chord whose mid-point is $(t, K - t)$

$$\frac{tx}{8} + \frac{(K-t)y}{2} = \frac{t^2}{8} + \frac{(K-t)^2}{2} \qquad \ldots\text{(i)}$$

$(2,-1)$ satisfy (i)

$\Rightarrow\ 5t^2 - (6+8K)t + 4\left(K^2 + K\right) = 0$

For two distinct chords, $D > 0$

$(6+8K)^2 - 4.4.5\left(K^2 + K\right) > 0$

$\Rightarrow\ 4K^2 - 4K - 9 < 0$

$K \in \left(\frac{1-\sqrt{10}}{2}, \frac{1+\sqrt{10}}{2}\right) \Rightarrow a + b = 1$

4.(4) Let $f(x)=ax^2+\frac{b}{x}-c \qquad f'(x)=2ax-\frac{b}{x^2}$ d

$$f'(x)=0,\ x=\left(\frac{b}{2a}\right)^{\frac{1}{3}} \Rightarrow a\left(\frac{b}{2a}\right)^{\frac{2}{3}}+\frac{b}{\left(\frac{b}{2a}\right)^{\frac{1}{3}}}\geq c \Rightarrow \frac{27ab^2}{c^3}\geq 4$$

5.(8) $x+y+z=1$

$(y+z)+(z+x)+(x+y)=2$

Let $y+z=A, z+x=B, \quad x+y=C$

$(1+x)(1+y)(1+z)=(B+C)(C+A)(A+B)$

$\Rightarrow (1+x)(1+y)(1+z)\geq 8(x+y)(x+z)(z+x)$ …..(i)

As, $(x+y)(y+z)(z+x)=(1-x)(1-y)(1-z)$

$=1-(x+y+z)+(xy+yz+zx)-xyz = xy+yz+zx-xyz$

$(x+y)(y+z)(z+x)=xyz\left[\frac{1}{x}+\frac{1}{y}+\frac{1}{z}-1\right] = xyz\left[x^{-1}+y^{-1}+z^{-1}-1\right]$

Now $\frac{x^{-1}+y^{-1}+z^{-1}}{3}\geq\left(\frac{x+y+z}{3}\right)^{-1}$

$\Rightarrow (x+y)(y+z)(z+x)\geq 8xyz$ …..(ii)

$\Rightarrow (1+x)(1+y)(1+z)\geq 8.8xyz$

$\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\left(1+\frac{1}{z}\right)\geq 64$

6.(7) $\frac{\cos^3\theta}{9a}=\frac{\sin^3\theta}{5b}=\lambda^3$

$\frac{9a}{\cos\theta}+\frac{5b}{\sin\theta}=56$

$\frac{9a}{\lambda(9a)^{\frac{1}{3}}}+\frac{5b}{\lambda(5b)^{\frac{1}{3}}}=56$

$\left[(9a)^{\frac{2}{3}}+(5b)^{\frac{2}{3}}\right]^3=(56\lambda)^3=(56)^3$

7.(4) Any point on the curve $xy = 4$ is $\left(2t, \frac{2}{t}\right)$

Slope of the tangent at $\left(2t, \frac{2}{t}\right)$ is $-\frac{1}{t^2}$ $\quad\therefore$ equation of tangent is $x + t^2 y - 4t = 0$

Since it is tangent to $x^2 + y^2 = 8$ also

$\therefore \quad \left|\frac{4t}{\sqrt{1+t^4}}\right| = 2\sqrt{2} \qquad$ i.e. $\quad t^2 = 1$

$\therefore$ equation of the tangent is $x + y = \pm 4$

Since the intercepts are positive $\qquad \therefore$ the tangent is $x + y = 4$

8.(8) Centre of circles c_1, c_2, c_3 are in A.P.

General term for abscissa of centres $= 1 + (n-1).3 = 3n - 2 \qquad \therefore$ centre of c_5 is (13, 0)

Radius of circles are in G.P.

$\therefore R_n = 1.2^{n-1} = 2^{n-1} \qquad \therefore R_3 = 4$ and centre of c_3 is (7, 0)

Tangents of circle c_3 intersect each other at (13, 0) equation of any line passing through (13, 0) is

$y - 0 = m(x - 13) \Rightarrow mx - y - 13m = 0$ now it will be required tangents if $\left|\frac{7m - 0 - 13m}{\sqrt{m^2+1}}\right| = 4$

$\Rightarrow 36m^2 = 16m^2 + 16 \Rightarrow 20m^2 = 16 \Rightarrow m \pm \frac{2}{\sqrt{5}}$

Let $m_1 = \frac{2}{\sqrt{5}}, m_2 = -\frac{2}{\sqrt{5}} \qquad \therefore 10|m_1 m_2| = 8$

9.(BC) $\because T_{r+1} = {}^{15}C_r \left(x^4\right)^{(15-r)} \left(x^{-3}\right)^r = {}^{15}C_r x^{60-7r}$

(A) for the term independent of *x*, $60 - 70r = 0 \Rightarrow r$ is not an integer.

$\therefore$ there is no term independent of *x*.

(B) *n* = 15 is odd

$\therefore$ ${}^nC_r$ will be maximum if $r = \frac{n-1}{2}$ or $r = \frac{n+1}{2}$ $\qquad$ i.e. $r = 7$ or $r = 8$

$\therefore$ binomial coefficient of 8th and 9th terms will be greatest

(C) for the coefficient of x^{32}; $60 - 7r = 32 \Rightarrow r = 4$

$\therefore$ coefficient of x^{32} is $= {}^{15}C_4$

for the coefficient. of $x^{-17}; 60 - 7r = -17; r = 11$

$\therefore$ coefficient of x^{-17} is $= {}^{15}C_{11} = {}^{15}C_4 \qquad \therefore$ (C) is correct.

(D) If $x = \sqrt{2}, T_{r+1} = {}^{15}C_r\, 2^{\frac{60-7r}{2}}$

$\therefore$ for rational terms $r = 0, 2, 4, 6, ... 14$

10.(BD) Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$\because$ it cuts $x^2 + y^2 = 4$ orthogonally

$\Rightarrow c = 4$

Moreover $-2g + 2f + 9 = 0$

($\because$ $(-g, -f)$ satisfy the given equation)

$\therefore$ $S \equiv x^2 + y^2 + 2gx + 2fy + 4 = 0$

$\Rightarrow$ $x^2 + y^2 + (2f + 9)x + 2fy + 4 = 0$

$\Rightarrow$ $(x^2 + y^2 + 9x + 4) + 2f(x + y) = 0$

It is of the form $S + \lambda P = 0$ and hence passes through the intersection of $S = 0$ and $P = 0$ which when solved give $(-1/2, 1/2)$, $(-4, 4)$.

11.(BD) $\dfrac{az+b}{z+1} = \dfrac{ax+b+aiy}{(x+1)+iy} = \dfrac{(ax+b+aiy)((x+1)-iy)}{(x+1)^2+y^2}$

$\therefore$ $\text{Im}\left(\dfrac{az+b}{z+1}\right) = \dfrac{-(ax+b)y+ay(x+1)}{(x+1)^2+y^2} \Rightarrow \dfrac{(a-b)y}{(x+1)^2+y^2} = y$

$\because$ $a - b = 1$

$\therefore$ $(x+1)^2 + y^2 = 1$ $\quad\therefore$ $x = -1 \pm \sqrt{1-y^2}$

12.(AD) Given, x_1 and x_2 are roots of $\alpha x^2 - x + \alpha = 0$.

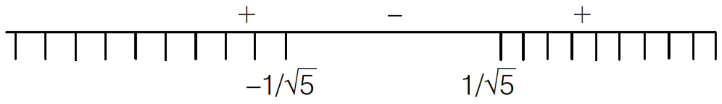
$\therefore$ $\quad x_1 + x_2 = \dfrac{1}{\alpha}$ and $x_1 x_2 = 1$

Also, $\quad |x_1 - x_2| < 1$

$\Rightarrow$ $\quad |x_1 - x_2|^2 < 1$ $\Rightarrow$ $(x_1 - x_2)^2 < 1$ $\quad$ or $\quad$ $(x_1 + x_2)^2 - 4x_1x_2 < 1$

$\Rightarrow$ $\quad \dfrac{1}{\alpha^2} - 4 < 1$ or $\dfrac{1}{\alpha^2} < 5$

$\Rightarrow$ $\quad 5\alpha^2 - 1 > 0$ or $(\sqrt{5}\,\alpha - 1)(\sqrt{5}\,\alpha + 1) > 0$



$\therefore$ $\quad \alpha \in \left(-\infty, -\dfrac{1}{\sqrt{5}}\right) \cup \left(\dfrac{1}{\sqrt{5}}, \infty\right)$ $\qquad$ …(i)

Also, $\quad D > 0$

$\Rightarrow$ $\quad 1 - 4\alpha^2 > 0$ or $\alpha \in \left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$ $\qquad$ …(ii)

From Eqs. (i) and (ii), we get

$\alpha \in \left(-\dfrac{1}{2}, \dfrac{-1}{\sqrt{5}}\right) \cup \left(\dfrac{1}{\sqrt{5}}, \dfrac{1}{2}\right)$

13.(ABD) Let $z=\alpha$ be a real root. Then $\alpha^3+(3+2i)\alpha+(-1+ia)=0$

$$\Rightarrow\ (\alpha^3+3\alpha-1)+i(a+2\alpha)=0 \qquad \Rightarrow\qquad \alpha^3+3\alpha-1=0 \text{ and } \alpha=-a/2$$

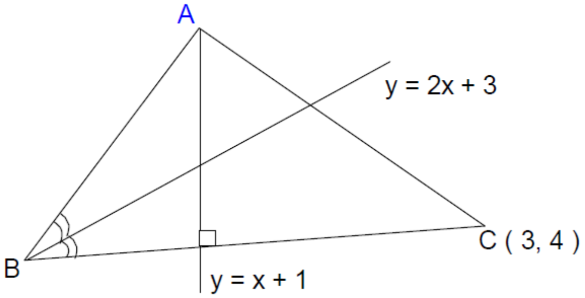
$$\Rightarrow\ -\frac{a^3}{8}-\frac{3a}{2}-1=0 \qquad \Rightarrow\qquad a^3+12a+8=0$$

Let $f(a)=a^3+12a+8 \quad \therefore\ f(-1)<0,\ f(0)>0,\ f(-2)<0,\ \ f(1)>0$ and $f(3)>0$

Hence, $a\in(-1,0)$ or $a\in(-2,1)$ *or* $a\in(-2,3)$

14.(AC) $|z|+|z-\cos\alpha-i\sin\alpha|=|z|+|z-e^{i\alpha}|\geq|z+(-z+e^{i\alpha})|=|e^{i\alpha}|=1 \Rightarrow |z|+|z-e^{i\alpha}|\geq 1$

15.(C) Equation of BC is $x+y=7$ so $B=\left(\frac{4}{3},\frac{17}{3}\right)$



Let slope of AB is m so, $\left|\frac{m-2}{1+2m}\right|=\left|\frac{2+1}{1-2}\right|$

$$m=-\frac{1}{7}$$

So equation of AB is $3x+21y-123=0$

So $A\equiv\left(\frac{17}{4},\frac{21}{4}\right)$ mid-point of $AB=\left(\frac{67}{24},\frac{131}{24}\right)$

Equation of median through C is $7x+y=25$

16.(C) $\dfrac{(1+x)^n+(1-x)^n}{2}={}^nc_0+{}^nc_2x^2+{}^nc_4x^4+\ldots\ldots \qquad \ldots.(i)$

$\dfrac{(1+ix)^n+(1-ix)^n}{2}={}^nc_0-{}^nc_2x^2+{}^nc_4x^4\ \ldots\ldots.. \qquad \ldots(ii)$

Adding (i) & (ii)

$$\frac{1}{2}\left[\frac{(1+x)^n+(1-x)^n}{2}+\frac{(1+ix)^n+(1-ix)^n}{2}\right]=\left[{}^nc_0+{}^nc_4x^4+\ldots..\right]$$

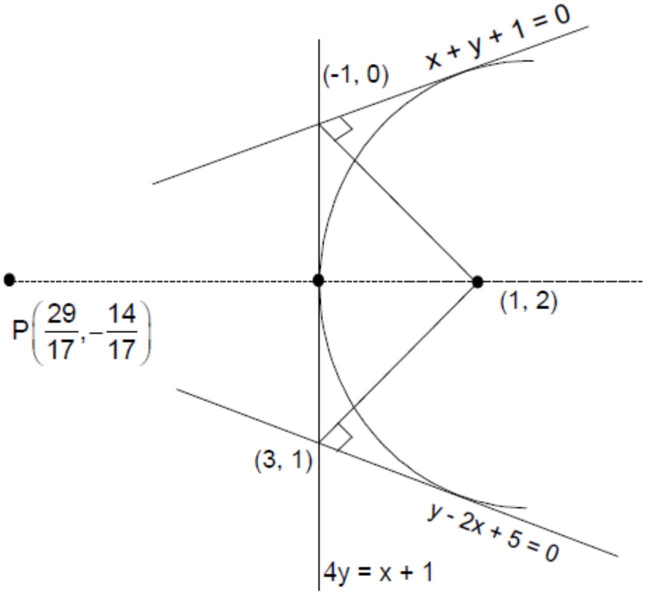
Put $x=1$, we get $\dfrac{1}{2}\left[\dfrac{2^n}{2}+\dfrac{(1+i)^n+(1-i)^n}{2}\right] \quad =\dfrac{1}{2}\left[2^{n-1}+\dfrac{2^{\frac{n}{2}}\cos n\frac{\pi}{4}}{2}\right]$

17.(D) Foot of perpendicular from focus on any tangent lies of tangent at the vertex.

Normal through given point be the axis of parabola,

$$y-2=-4(x-1)$$

$$4x+y=6$$



18.(B) $f_6(\cos\theta)=\cos 6\theta=2\cos^2 3\theta-1$

$$=2\left(4\cos^3\theta-3\cos\theta\right)^2-1=32\cos^6\theta-48\cos^4\theta+18\cos^2\theta-1$$